SYNOPTIC: Automatic Frequency-Domain Synthesis of Multiloop Control Systems, T. C. Coffey, Aerospace Corporation, El Segundo, Calif.; *AIAA Journal*, Vol. 8, No. 10, pp. 1791–1797.

Launch Vehicle and Controlled Missile Dynamics and Control; Spacecraft Attitude Dynamics and Control; Navigation, Control, and Guidance Theory

Theme

This paper presents the formulation of a computerized algorithm to facilitate the automatic synthesis of time invariant linear compensation for highly complex multiloop control systems. Performed in the frequency domain to attain desired open-loop frequency response characteristics, the algorithm is applicable equally to continuous-time systems in the S domain and sampled data systems in the W or Z domains. It is executed by a digital computer program, AUTO, which uses a gradient-search algorithm to select the coefficients of multiloop feedback compensation transfer functions. These coefficients provide an open-loop frequency response, optimum in the sense that its deviation from the one desired is minimal in the weighted least-square sense.

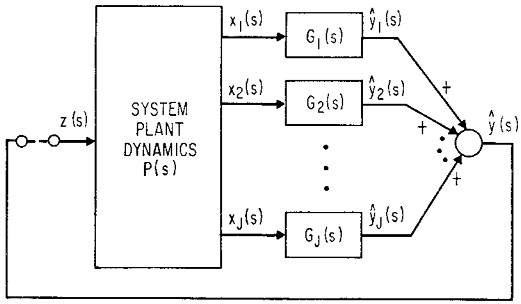

Fig. 1 Block diagram of a typical unforced, continuous time, multiloop control system.

Content

There exists a sizeable body of literature on the systematic analysis of complex linear dynamic systems. The problem treated in this paper is the frequency domain synthesis of such systems. The system structure treated is illustrated in Fig. 1.

The transfer functions are given by

$$G_j(s_l) = \left[\sum_{i=1}^{M_j} a_{ji}(s_l)^{i-1}\right] \Big/ \left[1 + \sum_{i=2}^{N_j} b_{ji}(s_l)^{i-1}\right]$$

where s_l is the Laplace variable corresponding to a particular frequency and $M_j - 1$ and $N_j - 1$ are, respectively, the numbers of zeros and poles of the jth transfer function.

The problem is one of selecting the coefficients of the transfer functions $G_j(s)$, so as to minimize a quadratic cost functional of the form

$$J = ||(\mathbf{y} - \hat{y})^T W^T W(\mathbf{y} - \hat{y})||$$

where $\mathbf{y}$ is a vector describing the desired open-loop frequency response of the system, $\hat{y}$ is the corresponding response of the designed system, and W is a weighting matrix. The uncompensated frequency responses $x_j(s)$ of the plant are assumed to be given. It will be noticed that J is in general a nonlinear function of the transfer function coefficients.

A gradient search algorithm is utilized to select the design parameters. The gradient is calculated analytically from expressions developed in the paper. The geometrical nature of the cost functions involved is discussed, and it is shown that a suitable method for avoiding convergence difficulties in minimizing the cost function involves the use of percentage, rather than absolute, changes in the transfer function coefficients.

The basic equations and organization of the synthesis program AUTO are presented. An illustration is given of the way in which AUTO is used to design W-plane compensation for a ballistic missile having a digital autopilot. Results for this case are presented. These results may be considered typical as AUTO has been used with great success in the design of both analog and digital autopilots for ballistic missiles. It should be noted, however, that AUTO is not particularized to ballistic missile problems and may be utilized to design any linear time invariant continuous or unirate sampled data system of the structure depicted in Fig. 1.